Real Number

TOPIC 1 : Euclids Division Lemma and Fundamental Theorem of Arithmetic

VERY SHORT ANSWER TYPE QUESTIONS

1. Explain why 13233343563715 is a composite number?

Ans : [Board Term-1, 2016 LGRKEGO]

The given number ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

2. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a+b)$.

Ans : [Board Term-1, 2014]

a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then least prime factor of $(a+b)$ is 2.

3. What is the HCF of the smallest composite number and the smallest prime number?

Ans :

The smallest prime number is 2 and the smallest composite number is $4=2^2$.
Hence, required HCF $(2^2,\ 2)=2$.

4. Calculate the HCF of $3^3\times5$ and $3^2\times5^2$.

Ans :

We have
$$3^3\times5 = 3^2\times5\times3$$
$$3^2\times5^2 = 3^2\times5\times5$$
$$\text{HCF }(3^3\times5,\ 3^2\times5^2) = 3^2\times5 = 9\times5 = 45$$

5. If HCF $(a,b)=12$ and $a\times b=1{,}800$, then find LCM (a,b).

Ans :

We know that
$$\text{HCF}(a,b)\times\text{LCM}(a,b) = a\times b$$
Substituting the values we have
$$12\times\text{LCM}(a,b) = 1800$$
or,
$$\text{LCM}(a,\ b) = \frac{1{,}800}{12} = 150$$

SHORT ANSWER TYPE QUESTIONS - I

6. Find HCF of the numbers given below:
$k,2k,3k,4k$ and $5k$,, where k is a Positive integer.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of $k,2k,3k,4k$ and $5k$, is k.

7. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1, 2012, Set-69]

We have
$$90 = 9\times10 = 2\times3^2\times5$$
and
$$144 = 16\times9 = 2^4\times3^2$$
$$\text{HCF} = 2\times3^2 = 18$$
$$\text{LCM} = 2^4\times3^2\times5 = 720$$

8. Using Euclid's algorithm, find the HCF of 240 and 288.

Ans : [Board Term-1, 2012, Set-35]

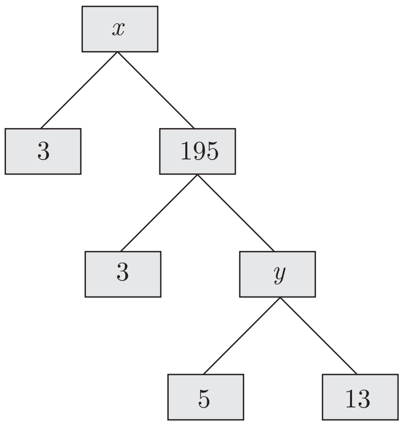
We have
$$240 = 228\times1+12$$
and
$$288 = 12\times19+0$$
Hence, HCF of 240 and 228 = 12

9. Given that HCF $(306,\ 1314)=18$. Find LCM $(306,\ 1314)$

Ans : [Board Term-1, 2013, FFC]

We have
$$\text{HCF }(306,\ 314) = 18$$
$$\text{LCM }(306,\ 1314) = ?$$
Let $a=306$ and $b=1314$, then we have
$$\text{LCM}(a,b)\times\text{HCF}(a,b) = a\times b$$
Substituting values we have
or,
$$\text{LCM}(a,b)\times18 = 306\times1314$$
or,
$$LCM\ (a,b) = \frac{306\times1314}{18}$$
$$LCM(306,1{,}314) = 22{,}338$$

10. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1, 2015, Set - WJQZQBN

We have $y = 5\times13 = 65$

and $x = 3\times195 = 585$

11. Explain why $(7\times13\times11)+11$ and $(7\times6\times5\times4\times3\times2\times1)+3$ are composite numbers.

Ans : [Board Term-1, 2012, Set-64]

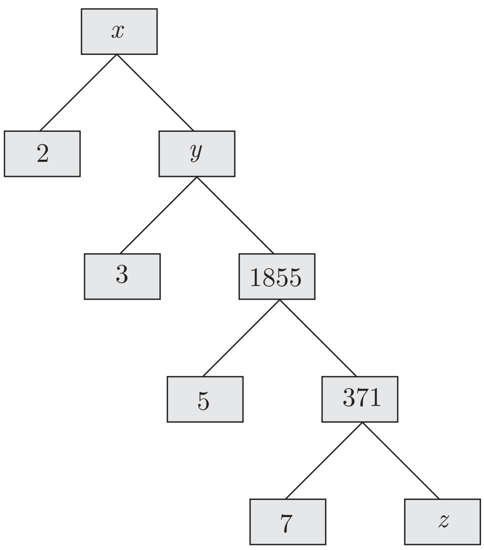
$$\begin{aligned}(7\times13\times11)+11 &= 11\times(7\times13+1)\\ &= 11\times(91+1)\\ &= 11\times92\end{aligned}$$

and

$$\begin{aligned}(7\times6\times5\times4\times3\times2\times1)+3 &= 3(7\times6\times5\times4\times2\times1+1)\\ &= 3\times(1681) = 3\times41\times41\end{aligned}$$

Since given numbers have more than two prime factors, both number are composite.

12. Complete the following factor tree and find the composite number x



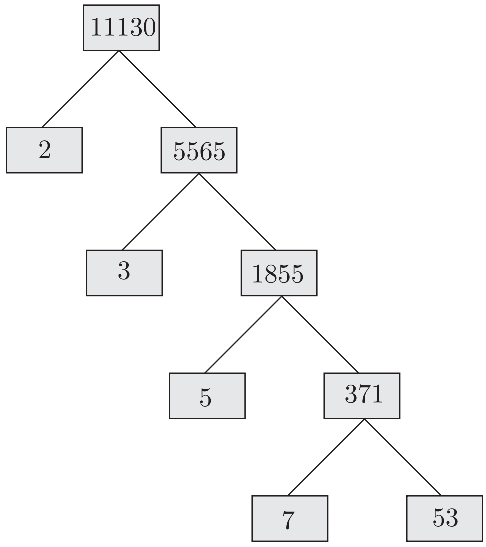
Ans : [Board Term-1, 2015, Set-DDE-M]

We have $z = \dfrac{371}{7} = 53$

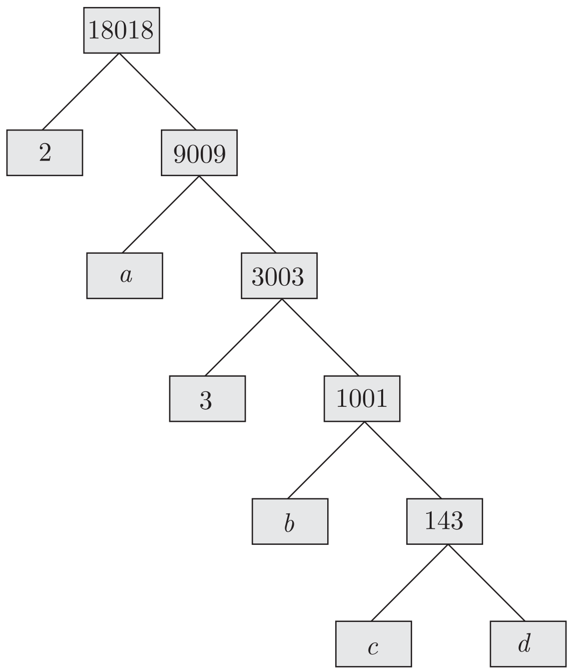
$y = 1855\times3 = 5565$

$x = 2\times y = 2\times5565 = 11130$

Thus complete factor three is as given below.



13. Find the missing numbers a, b, c and d in the given factor tree:



Ans : [Board Term-1, 2012, Set-52]

We have $a = \dfrac{9009}{3003} = 3$

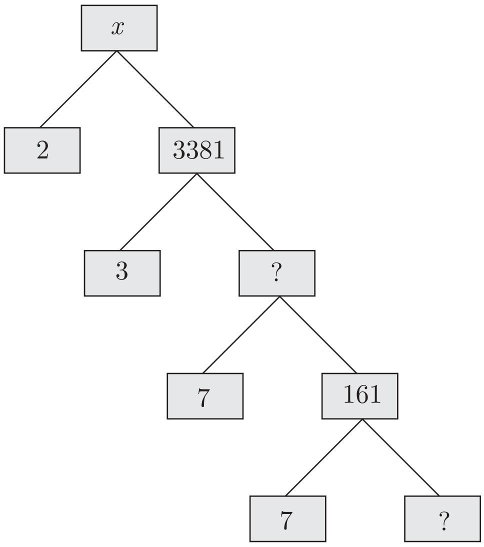
$b = \dfrac{1001}{143} = 7$

Since $143 = 11\times13$,

Thus $c = 11$ and $d = 13$ or $c = 13$ and $d = 11$

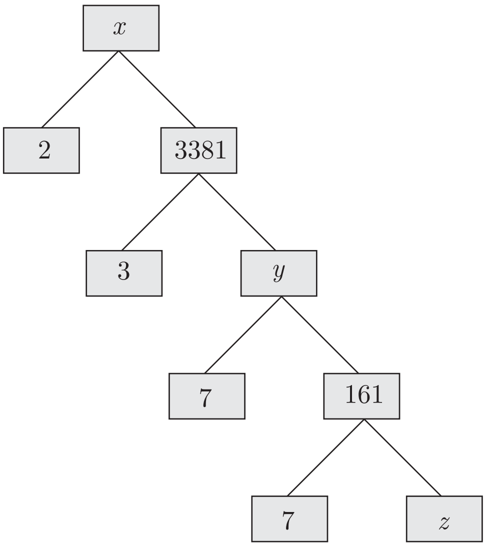
14. Complete the following factor tree and find the

composite number x.



Ans : [Term-1, 2015, Set - 44,], [Board term-2012, Set - 44]

We complete the given factor tree writing variable y and z as following.



We have $z = \frac{161}{7} = 23$

$y = 7 \times 161 = 1127$

Composite number, $x = 2 \times 3381 = 6762$

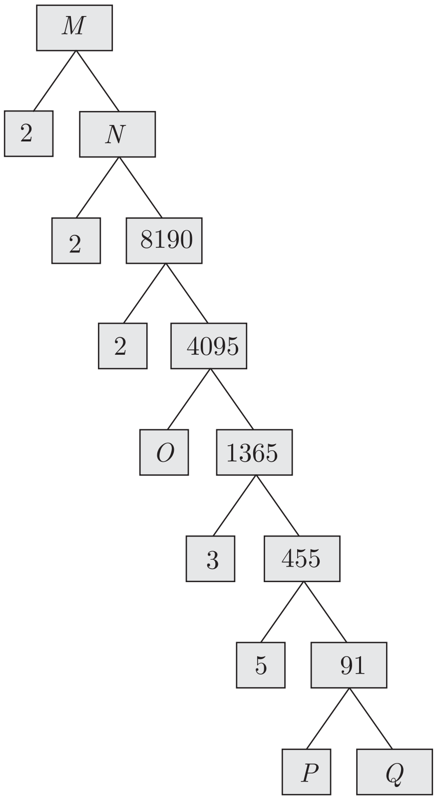
15. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Ans : [Board Term-1, 2016-17 Set; 193RQTQ, 2015, DDE-E]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1, involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned} 3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number} \end{aligned}$$

16. Complete the factor-tree and find the composite number M.



Ans : [NCERT]

We have $91 = P \times Q = 7 \times 13$

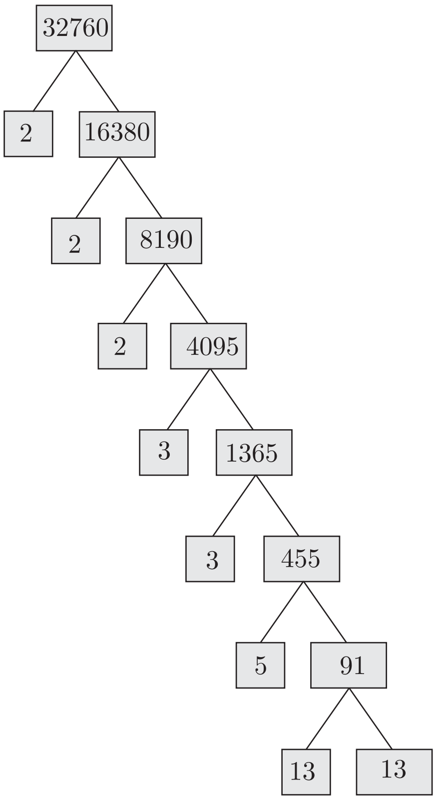
So $P = 7, Q = 13$ or $P = 13, Q = 7$

$O = \frac{4095}{1365} = 3$

$N = 2 \times 8190 = 16380$

Composite number $M = 16380 \times 2 = 32760$

Thus complete fact tree is shown below.



17. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.
[Term-1, 2015, WJQZQBN, [Term-1, 2016, WV98HN3]

Ans : [Term-1, 2015, WJQZQBN, [Term-1, 2016, WV98HN3]

We have $1200 = 12 \times 100$

$$= 4 \times 3 \times 4 \times 25$$

$$= 4^2 \times 3 \times 5^2$$

Here if we multiply by 3, then its square root will be a rational number because all power will be 2. Thus the required smallest natural number is 3. 2

18. Show that any positive even integer can be written in the form $6q, 6q+2$ or $6q+4$, where q is an integer.

Ans : [Board Term1, 2016 Set ORDAWEZ]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 6$, then $0 \le r < 6$ because $0 \le r < b$,

Thus $a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

Here $6q$, , $6q+2$ and $6q+4$ are divisible by 2 and so $6q$, $6q+2$ and $6q+4$ are even positive integers.

Hence a is always an even integer if

$$a = 6q,\ 6q+2,\ 6q+4$$

19. Show that any positive odd integer is of the form $4q+1$ or $4q+3$, where q is some integer.

Ans : [Board Term-1, Set-70,55][NCERT]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 4$, then $0 \le r < 4$ because $0 \le r < b$,

Thus $a = 4q, 4q+1, 4q+2, 4q+3$

Here we can see easily that $a = 4q$, $4q+2$ are even, as they are divisible by 2. Also $4q+1, 4q+3$ are odd, as they are not divisible by 2.

Thus any positive integer which has the form of $(4q+1)$ or $(4q+3)$ is odd.

20. Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Ans : [Board Term-1, 2012, Set-50]

LCM of two numbers should be exactly divisible by their HCF. Since, 15 does not divide 175, two numbers cannot have their HCF as 15 and LCM as 175.

21. Check whether 4^n can end with the digit 0 for any natural number n.

Ans : [Board Term-1, 2015, Set-FHN8MGD; NCERT]

If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

22. Show that 7^n cannot end with the digit zero, for any natural number n.

Ans : [Board Term-1, 2012, Set-63]

If the number 7^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 7^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $7^n = (1 \times 7)^n$ is 7. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 7^n. So, there is no natural number n for which 7^n ends with the digit zero. Hence 7^n cannot end with the digit zero.

23. Check whether $(15)^n$ can end with digit 0 for any $n \in N$.

Ans : [Board Term-1, 2012, Set-71]

If the number $(15)^n$, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of $(15)^n$ would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(15)^n = (3 \times 5)^n$ are 3 and 5. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(15)^n$. Since there is no prime factor 2, $(15)^n$ cannot end with the digit zero.

24. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the

dimensions of the room exactly.

Ans : [Board Term-1, 2016 Set ORDAWEZ]

Here we have to determine the HCF of all length which can measure all dimension.

Length, $l = 8m\,50cm = 850cm$ cm

$= 50 \times 17 = 2 \times 5^2 \times 17$

Breadth $b = 6m\,25\,cm = 625\,cm$

$= 25 \times 25 = 5^2 \times 5^2$

Height $h = 4m\,75cm = 475cm$

$= 25 \times 19 = 5^2 \times 19$

$\text{HCF}(l, b, h) = \text{HCF}\ (850, 625, 475)$

$= 5^2 = 25$

25. If two positive integers p and q are written as $p = a^2 b^3$ and $q = a^3 b$, where a and b are prime numbers than verify $LCM(p, q) \times HCF(q, q) = p.q$

Ans : [Sample Paper 2017]

We have $p = a^2 b^3 = a \times a \times b \times b \times b$

and $q = a^3 b = a \times a \times a \times b$

Now $LCM(p, q) = a \times a \times a \times b \times b \times b$

$= a^3 b^2$

and $HCF(p, q) = a \times a \times b$

$= a^2 b$

$LCM(p, q) \times HCF(p, q) = a^3 b^3 \times a^2 b$

$= a^5 b^4$

$= a^2 b^3 \times a^3 b$

SHORT ANS

26. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm.

Ans : [Board Term-1, 2016 Set MV98HN3]

We have $324 = 252 \times 1 + 72$

$252 = 72 \times 3 + 36$

$72 = 36 \times 2 + 0$

Thus $HCF(324, 252) = 36$

Now $180 = 36 \times 5 + 0$

Thus $HCF(36, 180) = 36$

Thus HCF of 180, 252, and 324 is 36.
Hence required number $= 999999 - 63 = 999936$ 1

27. Use Euclid division lemma to show that the square of any positive integer cannot be of the form $5m + 2$ or $5m + 3$ for some integer m.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$a = bq + r,\ 0 \le r < b$ and $q \in \omega$

Take $b = 5$, then $0 \le r < 5$ because $0 \le r < b$

Thus $a = 5q,\ 5q + 1,\ 5q + 2,\ 5q + 3$ and $5q + 4$,

Now $a^2 = (5q)^2 = 25q^2 = 5(5q^2) = 5m$

$a^2 = (5q + 1)^2 = 25q^2 + 10q + 1 = 5m + 1$

$a^2 = (5q + 2)^2 = 25q^2 + 20q + 4 = 5m + 4$

Similarly $a^2 = (5q + 3)^2 = 5m + 4$

and $a^2 = (5q + 4)^2 = 5m + 1$

Thus square of any positive integer cannot be of the form $5m + 2$ or $5m + 3$. 3

28. Show that numbers 8^n can never end with digit 0 of any natural number n.

Ans : [Board Term-1, 2015, Set-DDE-E][NCERT]

If the number 8^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 8^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(8)^n = (2^3)^n = 2^{3n}$ is 2. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(8)^n$. Since there is no prime factor 5, $(8)^n$ cannot end with the digit zero.

29. Find the HCF, by Euclid's division algorithm of the numbers 92690, 7378 and 7161.

Ans : [Board Term-1, 2013, Set LK-59]

By using Euclid's Division Lemma, we have

$92690 = 7378 \times 12 + 4154$

$7378 = 4154 \times 1 + 3224$

$4154 = 3224 \times 1 + 930$

$3224 = 930 \times 3 + 434$

$930 = 434 \times 2 + 62$

$434 = 62 \times 7 + 0$

$\text{HCF}\ (92690, 7378) = 62$

Now, using Euclid's Division Lemma on 7161 and 62, we have

$7161 = 62 \times 115 + 31$

$62 = 31 \times 2 + 0$

Thus HCF $(7161, 62) = 31$

Hence, HCF of 92690, 7378 and 7161 is 31.

30. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans : [Board Term-1, 2011, Set-66]

The required answer will be HCF of 144 and 90.

$144 = 2^4 \times 3^2$

$90 = 2 \times 3^2 \times 5$

$\text{HCF}(144,\ 90) = 2 \times 3^2 = 18$

Thus each stack would have 18 cartons.

31. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans : [Board Term-1, 2011, Set-44]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3\times3 = 3^2$$
$$12 = 2\times2\times3 = 2^2\times3$$
$$15 = 3 \times 5$$
$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$$
$$= 150 \text{ minutes}$$

The bells will toll next together after 180 minutes.

32. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number we have,

$$16 = 2\times2\times2\times2 = 2^4$$
$$36 = 2\times2\times3\times3 = 2^2\times3^2$$
$$\text{HCF}(16, 36) = 2\times2 = 4$$
$$\text{LCM } (16, 36) = 2^4\times3^2$$
$$= 16\times9 = 144$$

To check HCF and LCM by using formula

$$\text{HCF}(a, b)\times\text{LCM}(a, b) = a \times b$$

or, $$4\times144 = 16\times36$$
$$576 = 576$$

Thus $$\text{LHS} = \text{RHS}$$

33. Find the HCF and LCM of 510 and 92 and verify that HCF × LCM = Product of two given numbers.

Ans : [Board Term-1, 2011, Set-39]

Finding prime factor of given number we have,

$$92 = 2^2\times23$$
$$510 = 30 \times 17 = 2\times3\times5\times17$$
$$\text{HCF } (510, 92) = 2$$
$$\text{LCM } (510, 92 = 2^2\times23\times3\times5\times14$$
$$= 23460$$
$$\text{HCF } (510, 92) \times \text{LCM } (510, 92)$$
$$= 2\times23460 = 46920$$

Product of two numbers $= 510\times92 = 46920$

Hence, $\text{HCF}\times\text{LCM} = \text{Product of two numbers}$

34. The HCF of 65 and 117 is expressible in the form $65m–117$. Find the value of m. Also find the LCM of 65 and 117 using prime factorization method.

Ans : [Board Term-1, 2011, Set-40]

Finding prime factor of given number we have,

$$117 = 13\times2\times3$$
$$65 = 13\times5$$
$$\text{HCF}(117, 65) = 13$$
$$\text{LCM}(117, 65) = 13\times5\times3\times3 = 585$$
$$\text{HCF} = 65m - 117$$
$$13 = 65m–117$$
$$65m = 117 + 13 = 130$$
$$m = \frac{130}{6.5} = 2$$

35. Sho that any positive odd integer is of the form $6q+1, 6q+3$ or $6q+5$, where q is some integer.

Ans : [Board Term-1, 2011, Set-60]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 6$, then $0 \le r < 6$ because $0 \le r < b$,

Thus $$a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$$

Here $6q$, , $6q+2$ and $6q+4$ are divisible by 2 and so $6q$, $6q+2$ and $6q+4$ are even positive integers.
But $6q+1, 6q+3, 6q+5$ are odd, as they are not divisible by 2.
Thus any positive odd integer is of the form $6q+1, 6q+3$ or $6q+5$.

36. Show that exactly one of the number $n, n+2$ or $n+4$ is divisible by 3.

Ans : [Sample Paper 2017]

If n is divisible by 3, clearly $n+2$ and$n+4$ is not divisible by 3.
If n is not divisible by 3, then two case arise as given below.
Case 1: $n = 3k+1$

$$n+2 = 3k+1+2 = 3k+3 = 3(k+1)$$

and $$n+4 = 3k+1+4 = 3k+5 = 3(k+1)+2$$

We can clearly see that in this case $n+2$ is divisible by 3 and $n+4$ is not divisible by 3. Thus in this case only $n+2$ is divisible by 3.
Case 1: $n = 3k+2$

$$n+2 = 3k+2+2 = 3k+4 = 3(k+1)+1$$

and $$n+4 = 3k+2+4 = 3k+6 = 3(k+2)$$

We can clearly see that in this case $n+4$ is divisible by 3 and $n+2$ is not divisible by 3. Thus in this case only $n+4$ is divisible by 3.
Hence, exactly one of the numbers $n, n+2, n+4$, is divisible by 3.

LONG ANSWER TYPE QUESTIONS

37. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF × LCM of these numbers equal to the product of the given three numbers?

Ans :

Finding prime factor of given number we have,

$$378 = 2\times3^3\times7$$
$$180 = 2^2\times3^2\times5$$
$$420 = 2^2\times3\times7\times5$$
$$\text{HCF}(378, 180, 420) = 2\times3 = 6$$
$$\text{LCM}(378, 180, 420) = 2^2\times3^3\times5\times7$$
$$= 2^2\times3^3\times5\times7 = 3780$$
$$\text{HCF}\times\text{LCM} = 6\times3780 = 22680$$

Product of given numbers

$$= 378\times180\times420$$

$= 28576800$

Hence, HCF $\times$ LCM $\neq$ Product of three numbers.

38. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization by 3.

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of a unique combination of prime numbers.

OR

Every composite number can be expressed as the product powers of primes and this factorization is unique.

Finding prime factor of given number we have,

$$\begin{aligned} 2520 &= 20 \times 126 = 20 \times 6 \times 21 \\ &= 2^3 \times 3^2 \times 5 \times 7 \\ 10530 &= 30 \times 351 = 30 \times 9 \times 39 \\ &= 30 \times 9 \times 3 \times 13 \\ &= 2 \times 3^4 \times 5 \times 13 \\ \text{LCM}(2520, 10530) &= 2^3 \times 3^4 \times 5 \times 7 \times 13 \\ &= 294840 \end{aligned}$$

39. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

Ans : [Board Term-1, 2015, Set-WJQZBN]

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5.

That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

40. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

Ans : [Board Term-1, 2015, Set-WJQZBN]

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\begin{aligned} HCF &= 24 \\ LCM &= 540 \\ \frac{LCM}{HCF} &= \frac{540}{24} = 22.5 \text{ not an integer} \end{aligned}$$

41. Find the HCF of 256 and 36 using Euclid's Division Algorithm. Also, find their LCM and verify that HCF $\times$ LCM = Product of the two numbers.

Ans : [Board Term-1, 2015, Set-DDE-E]

By using Euclid's Division Lemma, we have

$$\begin{aligned} 256 &= 36 \times 7 + 4 \\ 36 &= 4 \times 9 + 0 \end{aligned}$$

Hence, the HCF of 256 and 36 is 4.

LCM :
$$\begin{aligned} 256 &= 2^8 \\ 36 &= 2^2 \times 3^2 \\ \text{LCM }(36, 256) &= 2^8 \times 3^2 = 256 \times 9 \\ &= 2304 \\ \text{HCF} \times \text{LCM} &= \text{Product of the two number} \\ 4 \times 2,304 &= 256 \times 36 \\ 9216 &= 9,216 \end{aligned}$$

Hence verified.

42. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

Required answer is the HCF of 990 and 945.
By using Euclid's Division Lemma, we have

$$\begin{aligned} 990 &= 945 \times 1 + 45 \\ 945 &= 45 \times 21 + 0 \end{aligned}$$

Thus HCF of 990 and 945 is 45. The fruit vendor should put 45 fruits in each basket to have minimum number of baskets.

43. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Ans : [Board Term-1, 2015, 2012, Set-48]

We have
$$\begin{aligned} n^3 - n &= n(n^2 - 1) \\ &= (n-1)n(n+1) \\ &= (n-1)n(n+1) \end{aligned}$$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

Let $a, a+1, a+2$ be any three consecutive integers.

Case I : $a = 3q$

If $a = 3q$ then,

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$

Product of two consecutive integers $(3q+1)$ and $(3q+2)$ is an even integer, say $2r$.

$$\begin{aligned} \text{Thus } a(a+1)(a+2) &= 3q(2r) \\ &= 6qr, \text{ which is divisible by 6.} \end{aligned}$$

Case II : $a = 3q+1$

If $a = 3q+1$ then

$$\begin{aligned} a(a+1)(a+2) &= (3q+1)(3q+2)(3q+3) \\ &= (2r)(3)(q+1) \\ &= 6r(q+1) \text{which is divisible by 6.} \end{aligned}$$

Case III : $a = 3q+2$

If $a = 3q+2$ then

$$a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$$
$$= 3(3q+2)(q+1)(3q+4)$$

Here $(3q+2)$ and $= 3(3q+2)(q+1)(3q+4)$

$= $ multiple of 6 every q

$= 6r$ (say)

which is divisible by 6. Hence, the product of three consecutive integers is divisible by 6 and $n^3 - n$ is also divisible by 3.

44. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

Ans : [Board Term-1, 2012 Set-25]

We have $\quad n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

Case 1 : $n = 2q$

If $n = 2q$ we have

$$n(n-1) = 2q(2q-1)$$
$$= 2m,$$

where $m = q(2q-1)$ which is divisible by 2.

Case 1 : $n = 2q+1$

If $n = 2q+1$, we have

$$n(n-1) = (2q+1)(2q+1-1)$$
$$= 2q(2q+1)$$
$$= 2m$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

45. Find HCF of 81 and 237 and express it as a linear combination of 81 and 237 i.e. HCF $(81, 237) = 81x + 237y$ for some x and y.

Ans : [Board Term-1, 2012 Set-35] [NCERT]

By using Euclid's Division Lemma, we have

$$237 = 81 \times 2 + 75 \quad ...(1)$$
$$81 = 75 \times 1 + 6 \quad ...(2)$$
$$75 = 6 \times 12 + 3 \quad ...(3)$$
$$6 = 3 \times 2 + 0 \quad ...(4)$$

Hence,HCF $(81, 237) = 3$.

In order to write 3 in the form of $81x + 237y$,

$3 = 75 - 6\times12$

$= 75 - (81 - 75\times1)\times12$ Replace 6 from (2)

$= 75 - 81\times12 + 75\times12$

$= 75 + 75\times12 - 81\times12$

$= 75(1+12) - 81\times12$

$= 75\times13 - 81\times12$

$= 13(237 - 81\times2) - 81\times12$ Replace 75 from (1)

$= 13\times237 - 81\times2\times13 - 81\times12$

$= 237\times13 - 81(26+12)$

$= 237\times13 - 81\times38$

$= 81\times(-38) + 237\times(13)$

$= 81x + 237y]$

Hence $x = -38$ and $y = 13$. These values of x and y are not unique.

46. Show that the square of any positive integer is of the forms $4m$ or $4m+1$, where m is any integer.

Ans : [Board Term-1, 2012 Set-39]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 4$, then $0 \le r < 4$ because $0 \le r < b$,

Thus $\quad a = 4q, 4q+1, 4q+2, 4q+3$

Case 1 : $a = 4q$

$$a^2 = (4q)^2 = 16q^2 = 4(4q^2)$$
$$= 4m$$

where $m = 4q^2$

Case 2 : $a = 4q+1$

$$a^2 = (4q+1)^2 = 16q^2 + 8q + 1$$
$$= 4(4q^2 + 2q) + 1$$
$$= 4m + 1$$

where $m = 4q^2 + 2q$

Case 3 : $a = 4q+2$

$$a^2 = (4q+2)^2$$
$$= 16q^2 + 16q + 4$$
$$= 4(4q^2 + 4q + 1)$$
$$= 4m$$

where $m = 4q^2 + 4q + 1$

Case 4 : $a^2 = (4q+3)^2 = 16q^2 + 24q + 9$

$$= 16q^2 + 24q + 8 + 1$$
$$= 4(4q^2 + 6q + 2) + 1$$
$$= 4m + 1$$

where $m = 4q^2 + 6q + 2$

From cases 1, 2, 3 and 4 we conclude that the square of any +ve integer is of the form $4m$ or $4m+1$.

47. Use Euclid's Division Lemma to show that the cube of any positive integer is of the form $9m$, $9m+1$, or $9m+8$, for some integer m.

Ans : [KVS, NCERT]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 3$, then $0 \le r < 3$ because $0 \le r < b$,

Thus $\quad a = 3q, 3q+1$, and $3q+2$

Case 1 : $a = 3q$

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3)$$

$= 9m$ where $m = 3q^3$

Case 2 : $a = 3q+1$

$$a^3 = (3q+1)^3$$
$$= 27q^3 + 9q(3q+1) + 1$$
$$= 9(3q^3 + 3q^2 + 1) + 1$$

or $a^3 = 9m+1$ where $m = 3q^3+3q^2+1$

Case 3 : $a = 3q+2$

$$\begin{aligned} a^3 &= (3q+2)^3 \\ &= 27d^3+18d(3d+2)+8 \\ &= 9(3q^3+6q^2+4q)+8 \end{aligned}$$

or $a^3 = 9m+8$ where $m = 3q^2+6q^2+4q$

From Case 1, 2 and 3, we conclude that the cube of any positive integer is of the form $9m, 9m+1$ or $9m+8$ for some integer m.

TOPIC 2 : Irrational Numbers, Terminating and Non-terminating, Recurring Decimals

VERY SHORT ANSWER TYPE QUESTIONS

1. What is the condition for the decimal expansion of a rational number to terminate? Explain with the help of an example.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

The decimal expansion of a rational number terminates, if the denominator of rational number can be expressed as $2^m 5^n$ where m and n are non negative integers and p and q both co-primes.

e.g. $$\frac{3}{10} = \frac{3}{2^1\times 5^1} = 0.3$$

2. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

Ans : [Board Term-1, 2016 Set LGRKEGO]

Since $$\frac{1}{7}\times\frac{7}{100} = \frac{1}{100} = 0.01.$$

Thus smallest rational number is $\frac{7}{100}$

3. What type of decimal expansion does a rational number has? How can you distinguish it from decimal expansion of irrational numbers?

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

A rational number has its decimal expansion either terminating or non-terminating, repeating An irrational numbers has its decimal expansion non-repeating and non-terminating.

4. Calculate $\frac{3}{8}$ in the decimal form.

Ans :

We have $$\begin{aligned} \frac{3}{8} &= \frac{3}{2^3} = \frac{2\times 5^3}{2^3\times 5^3} \\ &= \frac{375}{10^3} = \frac{375}{1{,}000} \\ &= 0.375 \end{aligned}$$

5. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Ans :

We have $$\begin{aligned} \frac{6}{1250} &= \frac{6}{2\times 5^4} = \frac{6\times 2^3}{2\times 2^3\times 5^4} \\ &= \frac{6\times 2^3}{2^4\times 5^4} = \frac{6\times 2^3}{(10)^4} \\ &= \frac{48}{10000} = 0.0048 \end{aligned}$$

Thus $\frac{6}{1250}$ will terminate after 4 decimal places.

6. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.

Ans : [Sample Paper 2017]

We have $$\frac{7}{75} = \frac{7}{3\times 5^2}$$

Since denominator of given rational number is not of form $2^m\times 5^n$, Hence, It is non-terminating decimal expansion.

SHORT ANSWER TYPE QUESTIONS - I

7. Show that $5\sqrt{6}$ is an irrational number.

Ans : [Board Term-1 2015, Set-CJEOQ]

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b\neq 0$; a and b are co-primes.

Now $$5\sqrt{6} = \frac{a}{b}$$

$$\sqrt{6} = \frac{a}{5b}$$

or, $$\sqrt{6} = \text{rational}$$

But, $\sqrt{6}$ is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number. 1

8. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m\times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division.

Ans : [Board Term-1, 2012, Set-67, NCERT Exemplar]

We have $$\begin{aligned} 500 &= 25\times 20 \\ &= 5^2\times 5\times 4 \\ &= 5^3\times 2^2 \end{aligned}$$

Here denominator is 500 which can be written as $2^2\times 5^3$.

Now decimal expansion,

$$\begin{aligned} \frac{257}{500} &= \frac{257\times 2}{2\times 2^2\times 5^3} = \frac{514}{10^3} \\ &= 0.514 \end{aligned}$$

9. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Ans : [K.V.S.]

We have $\sqrt{2} = \sqrt{\frac{200}{100}}$ and $\sqrt{3} = \sqrt{\frac{300}{100}}$

We need to find a rational number x such that

$$\frac{1}{10}\sqrt{200} < x < \frac{1}{10}\sqrt{300}$$

Choosing any perfect square such as 225 or 256 in between 200 and 300, we have

$$x = \sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{5}{3}$$

Similarly if we choose 256, then we have

$$x = \sqrt{\frac{256}{100}} = \frac{16}{10} = \frac{8}{5}$$

10. Write the rational number $\frac{7}{75}$ will have a terminating decimal expansion. or a non-terminating repeating decimal.

Ans : [Sample Question Paper 2017,-18]

We have $\frac{7}{75} = \frac{7}{3 \times 5^2}$

The denominator of rational number $\frac{7}{75}$ can not be written in form $2^m 5^n$ So it is non-terminating repeating decimal expansion.

SHORT ANSWER TYPE QUESTIONS - II

11. Express $\left(\frac{15}{4} + \frac{5}{40}\right)$ as a decimal fraction without actual division.

Ans : [Board Term-1, 2011, Set-74]

We have
$$\frac{15}{4} + \frac{5}{40} = \frac{15}{4} \times \frac{25}{25} + \frac{5}{40} \times \frac{25}{25}$$
$$= \frac{375}{100} + \frac{125}{1000}$$
$$= 3.75 + 0.125 = 3.875$$

12. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Ans : [Board Term-1, 2011, Set-A1][NCERT]

Let
$$x = 0.3\overline{178}$$
$$x = 0.3178178178$$
$$10,000x = 3178.178178...$$
$$10x = 3.178178....$$

Subtracting, $9990x = 3175$

or, $x = \frac{3175}{9990} = \frac{635}{1998}$

13. Prove that $\sqrt{2}$ is an irrational number.

Ans : [Board Term-1, 2011, Set-A1. NCERT]

Let $\sqrt{2}$ be a rational number.

Then $\sqrt{2} = \frac{p}{q}$,

where p and q are co-prime integers and $q \neq 0$ On squaring both the sides we have,

$$2 = \frac{p^2}{q^2}$$

or, $p^2 = 2p^2$

Since p^2 is divisible by 2, thus p is also divisible by 2.

Let $p = 2r$ for some positive integer r, then we have

$$p^2 = 4r^2$$
$$2q^2 = 4r^2$$

or, $q^2 = 2r^2$

Since q^2 is divisible by 2, thus q is also divisible by 2. We have seen that p and q are divisible by 2, which contradicts the fact that p and q are co-primes. Hence, our assumption is false and $\sqrt{2}$ is irrational.

14. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

Ans :

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $\sqrt{p} = \frac{m}{n}$,

where m and n are co-primes and $n \neq 0$.
Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$

or, $pn^2 = m^2$ ...(1)

Here p divides pn^2. Thus p divides m^2 and in result p also divides m.
Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2 q^2$$

or, $n^2 = pq^2$

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[$\because$ p is prime and p divides $n^2 \Rightarrow p$ divides n]
Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational. Hence, $\sqrt{p}$ is irrational.

15. Prove that $3 + \sqrt{5}$ is an irrational number.

Ans :

Assume that $3 + \sqrt{5}$ is a rational number, then we have

$$3 + \sqrt{5} = \frac{p}{q}, \quad q \neq 0$$
$$\sqrt{5} = \frac{p}{q} - 3$$
$$\sqrt{5} = \frac{p - 3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3 + \sqrt{5}$ is an irrational number.

16. Prove that $\sqrt{5}$ is an irrational number and hence show that $2 - \sqrt{5}$ is also an irrational number.

Ans : [Board Term-1, 2011, Set-60]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b},$$

(a,b are co-primes and $b \neq 0$)

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2-\sqrt{5}$ be rational equal to a, then we have

$$2-\sqrt{5} = a$$

$$2-a = \sqrt{5}$$

Since we have assume $2-a$ is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2-\sqrt{5}$ is irrational.

17. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, a and b are prime number then. Verify.

LCM $\times (p.q.) \times$ HCF $(p.q.) = pq$.

Ans : [Sample Question Paper 2017-18]

We have $p = a^2b^3 = a \times a \times b \times b \times b$

and $q = a^3b = a \times a \times a \times b$

Now $LCM(p,q) = a \times a \times a \times b \times b \times b$

$$= a^3b^2$$

and $HCF(p,q) = a \times a \times b$

$$= a^2b$$

$$LCM(p,q) \times HCF(p,q) = a^3b^3 \times a^2b$$

$$= a^5b^4$$

$$= a^2b^3 \times a^3b$$

$$= pq$$

LONG ANSWER TYPE QUESTIONS

18. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7+2\sqrt{3}$ is also an irrational number.

Ans : [Board Term-1, 2012, Set-DDE-M]

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

$(a, b$ are co-primes and $b \neq 0)$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7+2\sqrt{3}$ be rational equal to a, then we have

$$7+2\sqrt{3} = \frac{p}{q}$$

$q \neq 0$ and p and q are co-primes

$$2\sqrt{3} = \frac{p}{q} - 7 = \frac{p-7q}{q}$$

or $$\sqrt{3} = \frac{p-7q}{2q}$$

Here $p-7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7+2\sqrt{3}$ is irrational.

19. Show that there is no positive integer n, for which $\sqrt{n-1}+\sqrt{n-1}$ is rational.

Ans : [Board Term-1, 2012, Set-48]

Let us assume that there is a positive integer n for which $\sqrt{n-1}+\sqrt{n-1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers and $(q \neq 0)$.

$$\sqrt{n-1}+\sqrt{n-1} = \frac{p}{q} \quad ...(1)$$

or, $$\frac{q}{p} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})(\sqrt{n-1}-\sqrt{n+1})}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)}$$

or $$\frac{q}{p} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$$

$$\sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \quad ...(2)$$

Adding (1) and (2), we get

$$2\sqrt{n+1} = \frac{p}{q}+\frac{2q}{p} = \frac{p^2+2q^2}{pq}$$

...(3)

Subtracting (2) from (1) we have

$$2\sqrt{n-1} = \frac{p^2-2q^2}{pq}$$

...(4)

From (3) and (4), we observe that $\sqrt{n+1}$ and $\sqrt{n-1}$ both are rational because p and q both are rational. But it possible only when $(n+1)$ and $(n-1)$ both are perfect squares. But they differ by 2 and two perfect squares never differ by 2. So both $(n+1)$ and $(n-1)$ cannot be perfect squares, hence there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

HOTS QUESTIONS

20. Show that 571 is a prime number.

Ans :

Let $x = 571$

$$\sqrt{x} = \sqrt{571}$$

Now 571 lies between the perfect squares of $(23)^2 = 529$ and $(24)^2 = 576$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Here 571 is not divisible by any of the above numbers, thus 571 is a prime number.

21. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

Ans :

The required number is the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

$$\text{LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7$$
$$= 2520$$

22. An army contingent of 104 members is to march behind an army band of 96 members in a parade. The two groups are to march in the same number of columns in which they can march ?

Ans : [Board Term-1, 2012, Set-52]

Let the number of columns be x which is the largest number, which should divide both 104 and 96. It means x should be HCF of 104 and 96.

By using Euclid's Division Lemma, we have

$$104 = 96 \times 1 + 8$$
$$96 = 8 \times 12 + 0$$

Thus HCF of 104 and 96 is 8 and columns are required.

23. If d is the HCF of 30 and 72, find the value of x and y satisfying $d = 30x + 72y$.

Ans :

Using Euclid's Division Lemma, we have

$$72 = 30 \times 2 + 12 \quad ...(1)$$
$$30 = 12 \times 2 + 6 \quad ...(2)$$
$$12 = 6 \times 2 + 0 \quad ...(3)$$

Thus $\text{HCF}(30, 72) = 6$

Now $6 = 30 - 12 \times 2$

From (2)

$$6 = 30 - (72 - 30 \times 2) \times 2$$

From (1)

$$6 = 30 - 72 \times 2 + 30 \times 4$$
$$6 = 30(1 - 4) - 72 \times 2$$
$$6 = 30 \times 5 + 72 \times (-2)$$
$$6 = 30x + 72y$$

Thus $x = 5$ and $y = -2$ .Here x and y are not unique.

24. If HCF of 657 and 963 is expressible in the form of $657x + 963 \times (-15)$, find the value of x.

Ans :

Using Euclid's Division Lemma we have

$$963 = 657 \times 1 + 306$$
$$657 = 306 \times 2 + 45$$
$$306 = 45 \times 6 + 36$$
$$45 = 36 \times 1 + 9$$
$$36 = 9 \times 4 + 0$$
$$\text{HCF}(657, 963) = 9$$

Now $9 = 657x + 963 \times (-15)$

or, $657x = 9 + 963 \times 15$

$= 9 + 14445$

or, $657x = 14454$

or, $x = \dfrac{14454}{657} = 22$

25. Express the HCF/LCM of 48 and 18 as a linear combination.

Ans :

Using Euclid's Division Lemma, we have

$$48 = 18 \times 2 + 12 \quad (1)$$
$$18 = 12 \times 1 + 6 \quad (2)$$
$$12 = 6 \times 2 + 0$$

Thus $\text{HCF}(18, 48) = 6$

Now $6 = 18 - 12 \times 1$

From (2)

$$6 = 18 - (48 - 18 \times 2)$$

From (1)

$$6 = 18 - 48 \times 1 + 18 \times 2$$
$$6 = 18 \times (2 + 1) - 48 \times 1$$
$$6 = 18 \times 3 - 48 \times 1$$
$$6 = 18 \times 3 + 48 \times (-1)$$

Thus $6 = 18x + 48y$,

where $x = 3, y = -1$

Here x and y are not unique.

$$6 = 18 \times 3 + 48 \times (-1)$$
$$= 18 \times 3 + 48 \times (-1) + 18 \times 48 - 18 \times 48$$
$$= 18(3 + 48) + 48(-1 - 18)$$
$$= 18 \times 51 + 48 \times (-19)$$
$$6 = 18x + 48y,$$

where $x = 51, y = -19$